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Short Communication

Elimination of vibration localization: a mathematical justification

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Abstract

In this note, mistuned periodic structures are considered. Due to mistunings, some components of such structures may vibrate with small amplitudes, while some other components may vibrate with significantly large amplitudes. Such a behavior is known as vibration localization and is undesirable. In an earlier note, it was shown by the author that an effective passive technique to eliminate vibration localization in mistuned periodic structures is to add small components between the structure components. In this note, it is rigorously proved that the added components indeed eliminate vibration localization. The proof is established by using a dichotomy in the dynamics of the structure to which small components are added, and by applying the singular perturbation method.

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1. Introduction

Consider a periodic structure with n -coupled components, as that in Fig. 1. Let the normalized lengths of the structure components be the positive real numbers $\alpha_1, \alpha_2, \dots, \alpha_n$; let the normalized damping coefficient in the components be the positive real number γ ; and let the normalized coupling parameter of the components be the positive real number β . The dynamics of

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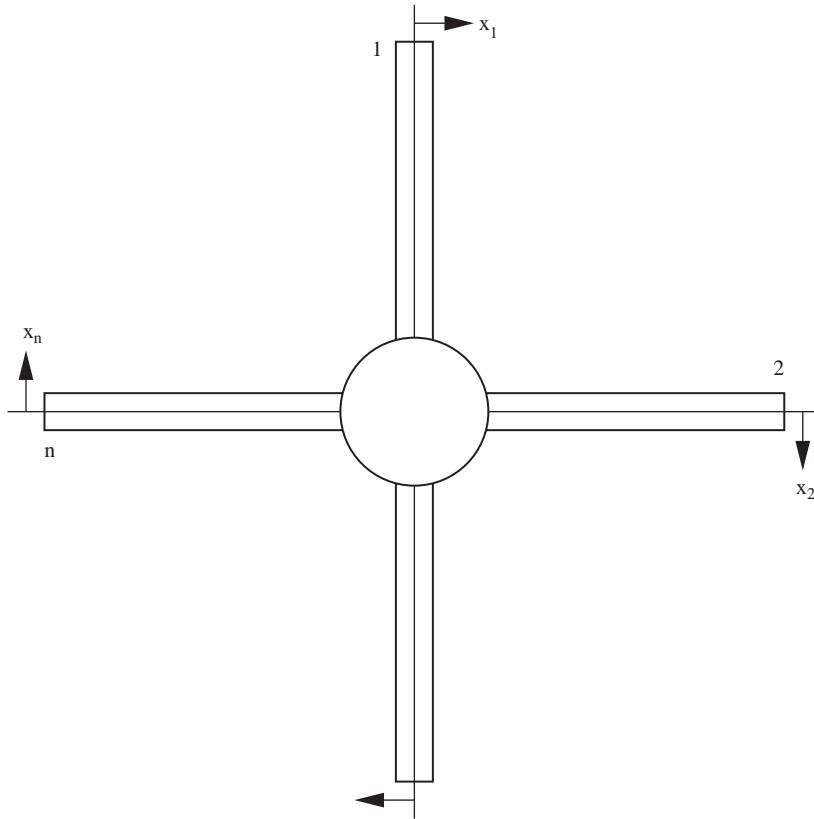


Fig. 1. A typical periodic structure having n components. Displacement of a component for an $i = 1, 2, \dots, n$ is denoted by x_i .

this structure can be represented by (see, e.g., Refs. [1,2])

$$M\ddot{\theta}(t) + \gamma M\dot{\theta}(t) + K\theta(t) = b_f f(t) \tag{1}$$

for all $t \geq 0$. In Eq. (1), the vector of angular displacements

$$\theta(t) = [\theta_1(t) \ \theta_2(t) \ \dots \ \theta_n(t)]^T \in \mathbb{R}^n \tag{2}$$

for all $t \geq 0$; the vectors of initial displacements and initial velocities are, respectively, $\theta(0) = \mathbf{0}_n$ and $\dot{\theta}(0) = \mathbf{0}_n$, where $\mathbf{0}_n$ denotes the zero vector in \mathbb{R}^n ; the input (influence) vector

$$b_f = [\alpha_1 \ \alpha_2 \ \dots \ \alpha_n]^T \in \mathbb{R}^n, \tag{3}$$

through which the scalar-valued input $t \rightarrow f(t)$ is applied to the structure; and the coefficient matrices are

$$M = \text{diag}[\alpha_1^3, \alpha_2^3, \dots, \alpha_n^3] \in \mathbb{R}^{n \times n}, \tag{4}$$

$$K = \begin{bmatrix} 1 + 2\beta & -\beta & 0 & 0 & \cdots & 0 & 0 & -\beta \\ -\beta & 1 + 2\beta & -\beta & 0 & \cdots & 0 & 0 & 0 \\ 0 & -\beta & 1 + 2\beta & -\beta & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \vdots & -\beta & 1 + 2\beta & -\beta \\ -\beta & 0 & 0 & 0 & \cdots & 0 & -\beta & 1 + 2\beta \end{bmatrix} \in \mathbb{R}^{n \times n}. \quad (5)$$

In a periodic structure, the lengths $\alpha_1, \alpha_2, \dots, \alpha_n$ should be equal to a desired value α_d . In reality, however, these lengths are different from each other and α_d , even only slightly. In this case, the structure is said to be *mistuned* and the differences between $\alpha_1, \alpha_2, \dots, \alpha_n$ and α_d are called mistunings. Mistunings are typically due to slight differences in the geometry and material properties of the structure components which are mainly introduced during manufacturing.

It is well known that small mistunings can cause significant differences in the dynamics of structure components, as reported in Refs. [1–6] and references therein. For instance, consider a mistuned structure under a harmonic input. In such a structure, at resonance frequencies, some components may vibrate with small amplitudes, while some other components may vibrate with significantly large amplitudes. This behavior is known as *vibration localization*.

It is desirable to eliminate vibration localization in mistuned periodic structures. In Ref. [2], an effective passive technique to eliminate vibration localization is presented. The elimination is achieved by adding small components between the structure components. Using a sensitivity matrix, it is shown in Ref. [2] that the added components successfully eliminate vibration localization.

In this note, without using the sensitivity matrix, it is rigorously proved that the added components to a mistuned periodic structure indeed eliminate vibration localization.

2. Elimination of vibration localization

In Ref. [2], a technique to eliminate vibration localization in mistuned periodic structures is presented. This technique is as follows. In a periodic structure, let there be n components numbered by $1, 2, \dots, n$; see Fig. 2. (Note that the numbering of components in the following is different from that in Ref. [2].) These components are called *principal components*. The normalized lengths of principal components are $\alpha_1, \alpha_2, \dots, \alpha_n$, and are desired to be equal to α_d .

Let the structure be augmented by adding n small components between principal components as shown in Fig. 2. The added components are numbered by $n + 1, n + 2, \dots, 2n$ and are called *auxiliary components*. The normalized lengths of auxiliary components are $\alpha_{n+1}, \alpha_{n+2}, \dots, \alpha_{2n}$, and are desired to be equal to $\alpha_s \ll \alpha_d$. This condition turns out to be crucially important in eliminating vibration localization in the structure.

A mathematical model representing the dynamics of the augmented structure is

$$\tilde{M}\ddot{\theta}(t) + \gamma\tilde{M}\dot{\theta}(t) + \tilde{K}\tilde{\theta}(t) = \tilde{b}_f f(t) \quad (6)$$

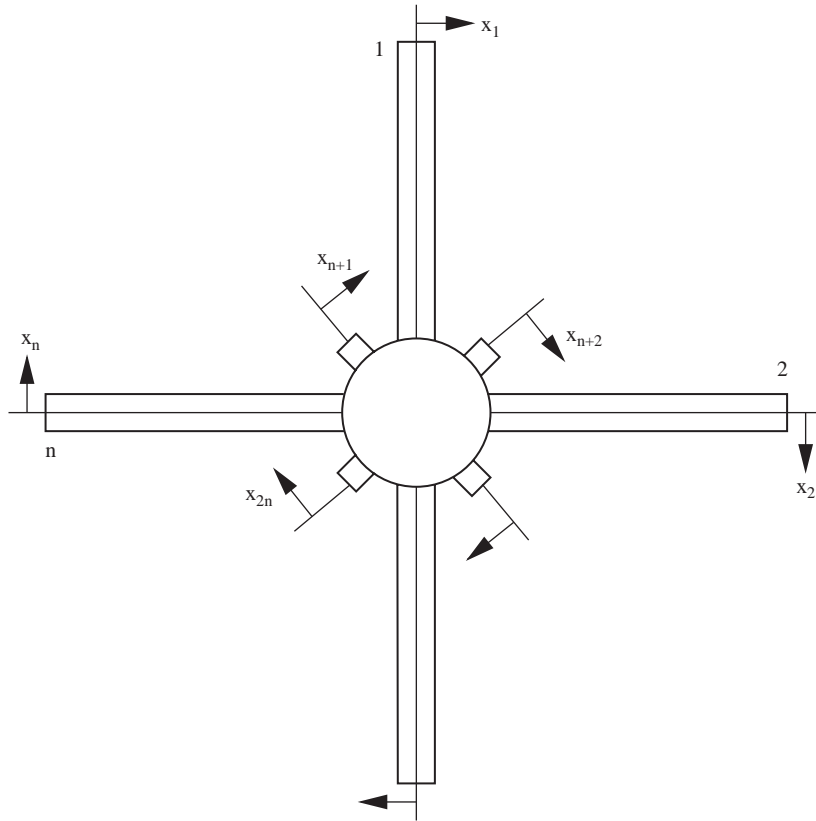


Fig. 2. A periodic structure is augmented by adding n small components between the structure components. The added components eliminate vibration localization.

for all $t \geq 0$. In Eq. (6), the vector of angular displacements

$$\tilde{\theta}(t) = \begin{bmatrix} \theta(t) \\ \theta_a(t) \end{bmatrix} \in \mathbb{R}^{2n} \tag{7}$$

for all $t \geq 0$, where $\theta(t)$ is that in Eq. (2) and

$$\theta_a(t) = [\theta_{n+1}(t) \ \theta_{n+2}(t) \ \cdots \ \theta_{2n}(t)]^T \in \mathbb{R}^n \tag{8}$$

is the vector of angular displacements of auxiliary components; the vectors of initial displacements and initial velocities are, respectively, $\tilde{\theta}(0) = \mathbf{0}_{2n}$ and $\dot{\tilde{\theta}}(0) = \mathbf{0}_{2n}$; the input (influence) vector

$$\tilde{b}_f = \begin{bmatrix} b_f \\ b_{f_a} \end{bmatrix} \in \mathbb{R}^{2n}, \tag{9}$$

where b_f is that in Eq. (3) and

$$b_{f_a} = [\alpha_{n+1} \ \alpha_{n+2} \ \cdots \ \alpha_{2n}]^T \in \mathbb{R}^n \tag{10}$$

is the input vector through which the input is applied to auxiliary components; the coefficient matrices are

$$\tilde{M} = \begin{bmatrix} M & 0 \\ 0 & M_a \end{bmatrix} \in \mathbb{R}^{2n \times 2n}, \tag{11}$$

where M is that in Eq. (4) and

$$M_a = \text{diag}[\alpha_{n+1}^3, \alpha_{n+2}^3, \dots, \alpha_{2n}^3] \in \mathbb{R}^{n \times n}, \tag{12}$$

and

$$\tilde{K} = \begin{bmatrix} K_{11} & K_{12} \\ K_{12}^T & K_{11} \end{bmatrix} \in \mathbb{R}^{2n \times 2n}, \tag{13}$$

where

$$K_{11} = (1 + 2\beta)I_n \in \mathbb{R}^{n \times n}, \tag{14a}$$

$$K_{12} = \begin{bmatrix} -\beta & 0 & 0 & 0 & \dots & 0 & 0 & 0 & -\beta \\ -\beta & -\beta & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & -\beta & -\beta & 0 & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \vdots & 0 & -\beta & -\beta & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & -\beta & -\beta \end{bmatrix} \in \mathbb{R}^{n \times n}, \tag{14b}$$

and I_n denotes the $n \times n$ identity matrix.

In studying vibration localization in mistuned periodic structures, in general, transfer functions from the applied input to structure displacements play an important role. For system (6), let $h_i(s)$ denote the transfer function from the input $f(\cdot)$ to the normalized displacement $x_i(\cdot) = \alpha_i \theta_i(\cdot)$ for an $i = 1, 2, \dots, 2n$. From Eq. (6), it follows that

$$\begin{bmatrix} h_1(s) \\ h_2(s) \\ \vdots \\ h_{2n}(s) \end{bmatrix} = \text{diag}[\alpha_1, \alpha_2, \dots, \alpha_{2n}] (\tilde{M}s^2 + \gamma \tilde{M}s + \tilde{K})^{-1} \tilde{b}_f. \tag{15}$$

To each transfer function $h_i(s)$ in Eq. (15), there corresponds an H_∞ -norm defined by

$$\|h_i\|_\infty := \max_{\omega \in \mathbb{R}} |h_i(j\omega)|, \tag{16}$$

where $j = \sqrt{-1}$. The norm $\|h_i\|_\infty$ corresponds to the global maximum of the Bode magnitude plot of the transfer function $h_i(s)$.

The occurrence of vibration localization in principal components due to mistunings in both principal and auxiliary components is easily determined when $\|h_1\|_\infty, \|h_2\|_\infty, \dots, \|h_n\|_\infty$ are known. If these norms do not differ much from each other, then vibration localization does not occur. Note that vibration localization in principal components is of primary interest. If vibration

localization in auxiliary components is to be studied, then H_∞ -norms of transfer functions corresponding to those components should be computed. As it will be shown in the next section, auxiliary components eliminate vibration localization in both principal and auxiliary components.

3. Mathematical justification

In this section, it is rigorously proved that auxiliary components indeed eliminate vibration localization in mistuned periodic structures. In other words, it will be shown that adding (small) auxiliary components to a mistuned structure results in an augmented structure in which H_∞ -norms of transfer functions corresponding to principle components are almost equal, so are those corresponding to auxiliary components.

System (6) can be written as

$$\begin{bmatrix} M & 0 \\ 0 & M_a \end{bmatrix} \begin{bmatrix} \ddot{\theta}(t) \\ \ddot{\theta}_a(t) \end{bmatrix} + \gamma \begin{bmatrix} M & 0 \\ 0 & M_a \end{bmatrix} \begin{bmatrix} \dot{\theta}(t) \\ \dot{\theta}_a(t) \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{12}^T & K_{11} \end{bmatrix} \begin{bmatrix} \theta(t) \\ \theta_a(t) \end{bmatrix} = \begin{bmatrix} M^{1/3} \mathbf{1}_n \\ M_a^{1/3} \mathbf{1}_n \end{bmatrix} f(t) \quad (17)$$

for all $t \geq 0$, where $\theta(0) = \theta_a(0) = \mathbf{0}_n$ and $\dot{\theta}(0) = \dot{\theta}_a(0) = \mathbf{0}_n$, and

$$\mathbf{1}_n := [1 \ 1 \ \dots \ 1]^T \in \mathbb{R}^n. \quad (18)$$

Vibration localization in system (17) is studied in the following. The conclusion to be reached is that if $0 < \alpha_s \ll \alpha_d$, then vibration localization in system (17) does not occur.

It is clear that the lengths of auxiliary components satisfy the following relation:

$$\alpha_{n+i} = \varepsilon \bar{\alpha}_{n+i} \quad (19)$$

for all $i = 1, 2, \dots, n$, where

$$\varepsilon := \frac{\alpha_s}{\alpha_d}, \quad (20a)$$

$$\bar{\alpha}_{n+i} = \frac{\alpha_d \alpha_{n+i}}{\alpha_s}. \quad (20b)$$

From Eq. (19), it follows that the matrix M_a in Eq. (12) satisfies

$$M_a = \varepsilon^3 \bar{M}_a, \quad (21)$$

where

$$\bar{M}_a = \text{diag}[\bar{\alpha}_{n+1}^3, \bar{\alpha}_{n+2}^3, \dots, \bar{\alpha}_{2n}^3]. \quad (22)$$

Using Eq. (22), system (17) can be written as

$$M \ddot{\theta}(t) + \gamma M \dot{\theta}(t) + K_{11} \theta(t) + K_{12} \theta_a(t) = M^{1/3} \mathbf{1}_n f(t), \quad (23a)$$

$$\varepsilon^3 \bar{M}_a \ddot{\theta}_a(t) + \varepsilon^3 \gamma \bar{M}_a \dot{\theta}_a(t) + K_{12}^T \theta(t) + K_{11} \theta_a(t) = \varepsilon \bar{M}_a^{1/3} \mathbf{1}_n f(t) \quad (23b)$$

for all $t \geq 0$, where $\theta(0) = \theta_a(0) = \mathbf{0}_n$ and $\dot{\theta}(0) = \dot{\theta}_a(0) = \mathbf{0}_n$.

It is noted that $0 < \varepsilon \ll 1$ since $\alpha_s \ll \alpha_d$. Small ε implies that there is a dichotomy in the dynamics of system (23): the vector $\theta(\cdot)$ evolves slowly in time, whereas the vector $\theta_a(\cdot)$ evolves fast. Due to

this dichotomy, system (23) can be studied by the singular perturbation method; see, e.g., Refs. [7–10] and references therein. According to this method, for sufficiently small ε , the dynamics of system (23) can be approximated by those of two subsystems. These subsystems are the slow and fast subsystems and are presented in the following.

3.1. Slow subsystem

The slow subsystem is an n -dimensional system obtained as follows. In Eq. (23b), first set $\varepsilon = 0$, and then solve for $\theta_a(\cdot)$ in terms of $\theta(\cdot)$. The result is

$$\theta_a(t) = -K_{11}^{-1} K_{12}^T \theta(t) \tag{24}$$

for all $t \geq 0$. Substituting $\theta_a(\cdot)$ into Eq. (23a), the representation of the slow subsystem is obtained as

$$M\ddot{\theta}(t) + \gamma M\dot{\theta}(t) + (K_{11} - K_{12}K_{11}^{-1}K_{12}^T)\theta(t) = M^{1/3}\mathbf{1}_n f(t) \tag{25}$$

for all $t \geq 0$, where $\theta(0) = \mathbf{0}_n$ and $\dot{\theta}(0) = \mathbf{0}_n$. Using matrices K_{11} and K_{12} given by Eq. (14), it follows that

$$K_{12}K_{11}^{-1}K_{12}^T = \frac{\beta^2}{1+2\beta} \begin{bmatrix} 2 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 1 \\ 1 & 2 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \vdots & 0 & 1 & 2 & 1 \\ 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 & 2 \end{bmatrix} \in \mathbb{R}^{n \times n}. \tag{26}$$

Since the normalized coupling parameter of components $0 < \beta \ll 1$, the matrix $K_{12}K_{11}^{-1}K_{12}^T$ is approximately equal to the null matrix, and hence it is neglected in Eq. (25). Thus, the representation of the slow subsystem can be written as

$$M\ddot{\theta}(t) + \gamma M\dot{\theta}(t) + K_{11}\theta(t) = M^{1/3}\mathbf{1}_n f(t) \tag{27}$$

for all $t \geq 0$, where $\theta(0) = \mathbf{0}_n$ and $\dot{\theta}(0) = \mathbf{0}_n$. Since M and K_{11} are diagonal matrices, system (27) is a set of n decoupled second order systems, one of which for an $i = 1, 2, \dots, n$ is

$$\alpha_i^3 \ddot{\theta}_i(t) + \gamma \alpha_i^3 \dot{\theta}_i(t) + (1 + 2\beta)\theta_i(t) = \alpha_i f(t) \tag{28}$$

for all $t \geq 0$, where $\theta_i(t) \in \mathbb{R}$ and $\theta_i(0) = 0$ and $\dot{\theta}_i(0) = 0$.

Having the simple representation in Eq. (28), the transfer function from $f(\cdot)$ to $x_i(\cdot) = \alpha_i \theta_i(\cdot)$ is readily obtained as

$$h_i(s) = \frac{\alpha_i^2}{\alpha_i^3 s^2 + \gamma \alpha_i^3 s + 1 + 2\beta} \tag{29}$$

for all $i = 1, 2, \dots, n$. From Eq. (29), it follows that

$$|h_i(j\omega)| = \frac{\alpha_i^2}{[(1 + 2\beta - \alpha_i^3\omega^2)^2 + \gamma^2\alpha_i^6\omega^2]^{1/2}}. \tag{30}$$

It can be easily verified that at the resonance frequency

$$\omega_i^* = \left(\frac{1 + 2\beta}{\alpha_i^3} - \frac{\gamma^2}{2} \right)^{1/2}, \tag{31}$$

the magnitude $|h_i(j\omega)|$ attains its maximum value given by

$$\|h_i\|_\infty = \frac{\alpha_i^{1/2}}{\gamma(1 + 2\beta - \gamma^2\alpha_i^3/4)^{1/2}} \tag{32}$$

for all $i = 1, 2, \dots, n$. Since α_i is close to α_d for all $i = 1, 2, \dots, n$, it follows that $\omega_1^*, \omega_2^*, \dots, \omega_n^*$ in Eq. (31) are almost equal, so are $\|h_1\|_\infty, \|h_2\|_\infty, \dots, \|h_n\|_\infty$ in Eq. (32).

3.2. Fast subsystem

The fast subsystem is an n -dimensional system obtained as follows. In system (23), set $t = \varepsilon^{3/2}\tau$. The result is

$$M \frac{d^2\theta(\tau)}{d\tau^2} + \varepsilon^{3/2}\gamma M \frac{d\theta(\tau)}{d\tau} + \varepsilon^3[K_{11}\theta(\tau) + K_{12}\theta_a(\tau)] = \varepsilon^3 M^{1/3} \mathbf{1}_n f(\tau), \tag{33a}$$

$$\bar{M}_a \frac{d^2\theta_a(\tau)}{d\tau^2} + \varepsilon^{3/2}\gamma \bar{M}_a \frac{d\theta_a(\tau)}{d\tau} + K_{12}^T \theta(\tau) + K_{11} \theta_a(\tau) = \varepsilon \bar{M}_a^{1/3} \mathbf{1}_n f(\tau), \tag{33b}$$

for all $\tau \geq 0$, where $\theta(0) = \theta_a(0) = \mathbf{0}_n$ and $d\theta(\tau)/d\tau = d\theta_a(\tau)/d\tau = \mathbf{0}_n$ at $\tau = 0$. Setting $\varepsilon = 0$ in Eq. (33a), it follows that $\theta(\tau) = \mathbf{0}_n$ for all $\tau \geq 0$, and hence Eq. (33b) can be written as

$$\bar{M}_a \frac{d^2\theta_a(\tau)}{d\tau^2} + \varepsilon^{3/2}\gamma \bar{M}_a \frac{d\theta_a(\tau)}{d\tau} + K_{11}\theta_a(\tau) = \varepsilon \bar{M}_a^{1/3} \mathbf{1}_n f(\tau) \tag{34}$$

for all $\tau \geq 0$, where $\theta_a(0) = \mathbf{0}_n$ and $d\theta_a(\tau)/d\tau = \mathbf{0}_n$ at $\tau = 0$. System (34) is the representation of the fast subsystem. Since \bar{M}_a and K_{11} are diagonal matrices, system (34) is a set of n decoupled second-order systems, one of which for an $i = 1, 2, \dots, n$ is

$$\bar{\alpha}_{n+i}^3 \frac{d^2\theta_{n+i}(\tau)}{d\tau^2} + \varepsilon^{3/2}\gamma \bar{\alpha}_{n+i}^3 \frac{d\theta_{n+i}(\tau)}{d\tau} + (1 + 2\beta)\theta_{n+i}(\tau) = \varepsilon \bar{\alpha}_{n+i} f(\tau) \tag{35}$$

for all $\tau \geq 0$, where $\theta_{n+i}(\tau) \in \mathbb{R}$ and $\theta_{n+i}(0) = 0$ and $d\theta_{n+i}(\tau)/d\tau = 0$ at $\tau = 0$. Setting $\tau = t/\varepsilon^{3/2}$ and using Eq. (19), it follows that Eq. (35) can be written as

$$\alpha_{n+i}^3 \ddot{\theta}_{n+i}(t) + \gamma \alpha_{n+i}^3 \dot{\theta}_{n+i}(t) + (1 + 2\beta)\theta_{n+i}(t) = \alpha_{n+i} f(t) \tag{36}$$

for all $t \geq 0$, where $\theta_{n+i}(0) = 0$ and $\dot{\theta}_{n+i}(0) = 0$.

Having the simple representation in Eq. (36), the transfer function from $f(\cdot)$ to $x_{n+i}(\cdot) = \alpha_{n+i}\theta_{n+i}(\cdot)$ is readily obtained as

$$h_{n+i}(s) = \frac{\alpha_{n+i}^2}{\alpha_{n+i}^3 s^2 + \gamma \alpha_{n+i}^3 s + 1 + 2\beta} \tag{37}$$

for all $i = 1, 2, \dots, n$. From Eq. (37), the magnitude $|h_{n+i}(j\omega)|$ can be obtained. This magnitude at the resonance frequency

$$\omega_{n+i}^* = \left(\frac{1 + 2\beta}{\alpha_{n+i}^3} - \frac{\gamma^2}{2} \right)^{1/2}, \tag{38}$$

attains its maximum value given by

$$\|h_{n+i}\|_\infty = \frac{\alpha_{n+i}^{1/2}}{\gamma(1 + 2\beta - \gamma^2 \alpha_{n+i}^3/4)^{1/2}} \tag{39}$$

for all $i = 1, 2, \dots, n$. Since α_{n+i} is close to α_s for all $i = 1, 2, \dots, n$, it follows that $\omega_{n+1}^*, \omega_{n+2}^*, \dots, \omega_{2n}^*$ in Eq. (38) are almost equal, so are $\|h_{n+1}\|_\infty, \|h_{n+2}\|_\infty, \dots, \|h_{2n}\|_\infty$ in Eq. (39).

Remarks. Having the slow and fast subsystems, several conclusions can be drawn:

(1) Since $\alpha_s \ll \alpha_d$, the H_∞ -norms in Eqs. (32) and (39) satisfy the following inequality:

$$\|h_{n+k}\|_\infty \ll \|h_i\|_\infty \tag{40}$$

for any $i, k = 1, 2, \dots, n$.

(2) For lightly damped structures, i.e., when $\gamma \ll 1$, it follows that

$$\|h_i\|_\infty = \frac{\alpha_i^{1/2}}{\gamma(1 + 2\beta)^{1/2}}, \quad \|h_{n+k}\|_\infty = \frac{\alpha_{n+k}^{1/2}}{\gamma(1 + 2\beta)^{1/2}}, \tag{41a}$$

$$\frac{\|h_{n+k}\|_\infty}{\|h_i\|_\infty} \approx \varepsilon^{1/2}, \tag{41b}$$

for any $i, k = 1, 2, \dots, n$.

(3) For lightly damped structures, the resonance frequencies of the slow and fast subsystems satisfy the following relation:

$$\frac{\omega_{n+k}^*}{\omega_i^*} \approx \left(\frac{\alpha_i}{\alpha_{n+k}} \right)^{3/2} \approx \frac{1}{\varepsilon^{3/2}} \gg 1 \tag{42}$$

for any $i, k = 1, 2, \dots, n$.

Having the slow and fast subsystems given by Eqs. (28) and (36), respectively, conclusions are drawn regarding the augmented structure, using the singular perturbed method. According to this method, for sufficiently small ε , the dynamics of the $2n$ -dimensional augmented structure in Eq. (23) can be approximated by those of the n -dimensional slow and fast subsystems over large and small time scales, respectively. Therefore, the transfer functions of principal (respectively, auxiliary) components can be approximated at low (high) frequencies by those of the slow (fast)

subsystem given by Eq. (29) (Eq. (37)). From Eq. (32) or Eq. (41a) for lightly damped structures, it is evident that H_∞ -norms of transfer functions of the slow subsystem are almost equal; hence, so are those of the transfer functions of principal components. That is, H_∞ -norms of transfer functions of principal components are insensitive to mistunings. In other words, vibration localization does not occur in principal components. It is also concluded that: (i) H_∞ -norms of the transfer functions of auxiliary components are almost equal to those in Eq. (39) or Eq. (41a) for lightly damped structures; (ii) by inequality (40) or Eq. (41b) for lightly damped structures, these norms are smaller than those of the transfer functions of principal components. Furthermore, by inequality (42), the resonance frequencies of the transfer functions of auxiliary components are much higher than those of principal components. Thus, (i) vibration localization does not occur in auxiliary components; (ii) amplitudes of vibration of such components are small, in particular, at low frequencies.

4. Conclusions

An effective passive technique to eliminate vibration localization in mistuned periodic structures is presented in Ref. [2]. In this reference, by computing a sensitivity matrix, it is shown that if small components are added between structure components, then vibration localization is eliminated in the structure. In this note, without using the sensitivity matrix, it is rigorously proved that the added components indeed eliminate vibration localization. The proof is established by using a dichotomy in the dynamics of the mistuned structure to which the small components are added, and by applying the singular perturbation method.

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